

# E pur si muove!

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## Abstract

We comment a debate on quantum physics in Slovak TV in which several physicists advocated a point of view that "an electron does not move around the nucleus".

This note records some musings about the quantum world which could perhaps help beginners in making first steps in quantum mechanics. It is principally addressed to students of Bratislava University but it may be of some interest more generally, e.g. to people fascinated by real or apparent paradoxes of the quantum world. In particular, on the example below, I point out disagreements between me and some teachers from Bratislava University about the interpretation of some basic concepts of quantum mechanics

In what follows, I shall summarise and comment a part of Štefan Hríb's TV show "The evening under the lamp" which is mainly devoted to topical political issues in Slovakia, but that particular evening it was devoted to "The smallest parts of the matter" [1]. Among invited guests there was a teacher of quantum field theory at Bratislava University Martin Mojžiš, two authors of a textbook on quantum mechanics Vladimír Černý and Ján Pišút and an experimental physicist Juraj Bracíník. Among other topics, it was discussed a question whether an electron moves around the nucleus. At some point Martin Mojžiš said:

*"... when the electron is in the ground state then it is in it and it is all the time in this state forever. If the atom did not interact with anything else it is in this state forever".*

The TV host Štefan Hríb then asked a truly outstanding question for a layman, in which he has shown his remarkable talent, intelligence and intuition. Indeed, in accordance with the very principles of the quantum theory (at least as I understand them), he half-stated half-asked:

*"This means that the electron moves in the same way forever?"*

To my big surprise Martin Mojžiš has disagreed with this interpretation and he objected:

*"Moves" is a wrong word, in this case misleading. It is forever in the same state. "*

When Štefan Hríb's opened his mouth to protest Martin Mojžiš did not let him speak and insisted:

*"Moves" is a misleading word. OK, it can be used but it obscures much more than it elucidates.*

Then all remaining physicists have supported the point of view of Martin Mojžiš, inspite of the fact that Štefan Hríb, visibly confused, was fighting hard. At the very end the TV host said reluctantly :

*"So we have abandoned the idea that the electron is moving around the nucleus..."*

Then he made a tiny break as if he expected a late sign of disapproval from the present physicists against such a "blasphemy". However, the experts did not react so Štefan Hríb continued:

*"Well, it is quite sad to tell the truth..."*

Again an infinitesimal break but the scientists remained inflexible. The TV host, visibly disappointed, then gave up:

*"Ok then..."*

Honestly, at the very beginning I did not understand why the colleagues from Bratislava were advocating such a strange point of view. Then it came into my mind that, perhaps, they do not understand the fact that if the electron remains all the time in the same state it does not mean at all that it is not moving!

To explain it, consider a free particle on a line or, better, on a large circle to avoid problems with normalisation of the wave function. Take the wave function of the particle proportional to  $e^{ip_1x}$  for some non-zero momentum  $p_1$  so the state described by this wave function REMAINS THE SAME all the time. However, it is obvious that the state  $e^{ip_1x}$  describes the MOVING particle since it is in the eigenstate of the momentum operator! The solution of this "paradox" is precisely that proposed by the TV host Štefan Hríb: the state of the free particle does not change with time since the particle moves all the time in the same way!

A similar analysis applies for a rotational motion. Although Martin Mojžiš was speaking about the ground state of the hydrogen atom (presumably with "spinless" electron), where there is no angular motion, the other physicists did have a possibility to clarify things by saying something like: *"The operator of angular momentum commutes with the Coulomb Hamiltonian therefore there are plenty excited angular momentum eigenstates which do not evolve with time but still they clearly describe the rotational motion of the electron around the nucleus."*

After all, even in the ground state the electron does move albeit in the radial way. To see that, it is sufficient to argue that the mean value of the kinetic energy in the ground state does not vanish. This can be seen without any calculation since the normalizability of the bounded ground state makes possible the integration "per partes" and the mean value of the kinetic energy  $\langle T \rangle$  is thus proportional to

$$\langle T \rangle \sim \int \nabla\psi^* \nabla\psi.$$

Obviously, this is a strictly positive quantity for the (nonconstant) ground state wave function  $\psi$ .

I think that the things can be made even clearer if we consider the classical limit of the quantum mechanical concept of the "state". Indeed, following the folkloric knowledge (nevertheless not mentioned in the textbook of Vladimír Černý and Ján Pišút), the classical limit of

the quantum state is a maximally isotropic submanifold of the classical phase space<sup>1</sup>. If we view the classical state as a point of the phase space, we observe that the classical limit of the quantum state is a collection of the classical states. If we consider classical Hamiltonian equations of motion, it may very well happen, that solutions with initial values on the maximally isotropic submanifold will remain contained in this submanifold for all subsequent times. Such maximally isotropic submanifold may be then called "stationary" and, upon quantization, it gives rise to a stationary quantum state which does not evolve with time. However, the classical motion within the stationary maximally isotropic submanifold is still present and it is present also in the corresponding quantum stationary state in the subtle way mentioned above.

**Example:**

Consider a classical free particle in one dimension. The coordinates of the phase space are  $p$  and  $q$  and the symplectic form is  $dp \wedge dq$ . Consider a constant number  $p_1$ . On the one-dimensional submanifold defined by the relation  $p = p_1$  the symplectic form vanishes therefore this submanifold is maximally isotropic. It is also stationary since the solution of the free equations (with the Hamiltonian  $H = p^2/2$ ) reads  $p(t) = p_1$ ,  $q(t) = q_1 + p_1 t$  therefore it respects all the times the constraint  $p = p_1$ . The quantization of the stationary maximally isotropic submanifold  $p = p_1$  is the wave function  $e^{ip_1 x}$  mentioned above and it encodes in it the free motion  $q(t) = q_1 + p_1 t$  in a subtle way, that is,  $e^{ip_1 x}$  is the eigenstate of the momentum operator with the eigenvalue  $p_1$ .

## References

- [1] [www.youtube.com/watch?v=PbLFWzeL-KM](http://www.youtube.com/watch?v=PbLFWzeL-KM), discussion between 1:52:00 and 1:56:30

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<sup>1</sup>Maximally isotropic submanifold of a  $2n$ -dimensional phase space is an  $n$ -dimensional submanifold on which the symplectic form vanishes.